Name: _____ Class: _____ Date: _____ ID: A

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Chapter Three Assignment

Short Answer

1. Why do roller coasters use clothoid loops instead of circular loops? Show mathematically.
2. Explain why "centrifugal" forces are fictitious forces.
3. Sketch two graphs. The first graph should illustrate the relationship between the force of gravity Earth exerts on objects and the mass of the objects. The second graph should show the variation of the force of gravity a pair of objects exert on one another and the separation distance between the objects' centres of mass.
4. Describe how the acceleration of gravity on Earth's surface would be different if both its mass and radius were twice their present values. Provide reasons for your answer. For full marks, do not use the earth's mass and radius to show the acceleration due to gravity.
5. Describe the limitations of the law of universal gravitation.

Problem

6. On a midway ride called *The Round Up*, participants climb into a cylindrical apparatus and stand upright with their backs against the wall of the cylinder. The apparatus begins to spin, first in the horizontal plane and then tipping into the vertical plane so that the riders' bodies are parallel to the ground below. Consider a rider of mass 60.0 kg. The radius of the apparatus is 8.0 m and it spins with a period of 4.0 s.
   (a) Determine the centripetal force acting on the person when the apparatus is still in the horizontal plane. Provide the appropriate free-body diagram and state the force that supplies the centripetal force in this situation.
   (b) Draw the free-body diagram of the person at the bottom of the apparatus when spinning in the vertical plane and determine how heavy the person feels at that position (i.e., what is the value of the normal force at that position?).
   (c) What minimum speed must the person be moving with in order to keep from falling away from the side of the cylinder when rotating in the vertical plane? State where in the circle this would occur and draw the appropriate free-body diagram.
7. A 2.0×10^2-g mass is tied on the end of a 1.6 m long string and whirled around in a circle that describes a vertical plane.
   (a) What is the minimum frequency of rotation required to keep the mass moving in a circle?
   (b) Calculate the maximum tension in the string at this frequency.
8. A 0.50-g insect rests on a compact disc at a distance of 4.0 cm from the centre. The disc's rate of rotation varies from 3.5 Hz to 8.0 Hz in order to maintain a constant data sampling rate.
   (a) What are the insect's minimum and maximum centripetal accelerations during its rotation around the disc?
   (b) What is the minimum value of the coefficient of static friction that would prevent the insect from slipping off the disc at the slowest rotation rate?
9. A pilot of mass 75 kg takes her plane into a dive, pulling out of it along a circular arc as she nears the ground. If the plane is flying at 1.5×10^2 km/h along the arc, what is its radius such that the pilot feels four times heavier than normal? Provide an appropriate free-body diagram.

10. A pilot of mass 60.0 kg is flying her plane in a vertically oriented circular loop. Just at the bottom of the loop, the plane's speed is 1.8×10^2 km/h and the pilot feels exactly four times as heavy as she normally does.
(a) What is the radius of the loop?
(b) At what speed must she be flying at the top of the loop in order to feel weightless?

11. An object of mass 6.0 kg is whirled around in a vertical circle on the end of a 1.0 m long string with a constant speed of 8.0 m/s. Include a free-body diagram for each of the following questions:
(a) Determine the maximum tension in the string, indicating the position of the object at the time the maximum tension is achieved.
(b) What is the minimum speed the object could be rotated with and maintain a circular path?
(c) If the object is rotated with the same speed (8.0 m/s) on a horizontal surface, what is the tension in the string if the string is parallel to the surface?

12. A flea stands on the end of a 1.0 cm long sweep second hand of a clock that rests horizontally on a table. What is the minimum coefficient of static friction which would allow the flea to stay there without slipping? Include an appropriate free-body diagram.

13. A ball of mass 4.0 kg is attached to the end of a 1.2 m long string and whirled around in a circle that describes a vertical plane.
(a) What is the minimum speed that the ball can be moving at and still maintain a circular path? Provide a free-body diagram.
(b) At this speed, what is the maximum tension in the string? Provide another free-body diagram.
(c) If the ball is rotated in a horizontal circle at the same speed with the end of the string held above the head, what angle does the string make with the horizontal?

14. What force does Earth exert on a 80.0-kg astronaut at an altitude equivalent to 2.5 times Earth's radius?

15. A planet has a mass of 2.5 times that of Earth and a radius 1.2 times Earth's radius. How much would a 60.0-kg person weigh at the planet's surface?

16. The gravitational field strength at the surface of a planet is 3.4 N/kg. If the planet's mass is 7.2×10^{22} kg, what is its radius?

17. A satellite orbits Earth at an altitude of 325 km above the planet's surface. What is its orbital period? Express your answer in minutes. ($r_E = 6.38 \times 10^6$ m, $M_E = 5.98 \times 10^{24}$ kg)

18. An Earth satellite has an orbital period of 3.2 h. What is its orbital radius?
($M_E = 5.98 \times 10^{24}$ kg)

19. A satellite has an orbital speed of 4.2×10^3 m/s. What is its altitude above Earth's surface?
($M_E = 5.98 \times 10^{24}$ kg, $r_E = 6.38 \times 10^6$ m)

Chapter Three Assignment
Answer Section

SHORT ANSWER

1. The speed of a roller coaster will decrease as it moves up a loop and increase on the way down. For a circular loop (of constant radius), the centripetal acceleration will decrease steadily as the roller coaster climbs the loop and increase steadily as it descends. The resulting centripetal force acting on passengers changes accordingly. The apparent centrifugal force that passengers feel changes as well. Designers of roller coasters have found that this results in an unpleasant ride for many passengers and may even lead to injury. When the clothoid loop is used (changing radius), the radius of the loop steadily decreases as the roller coaster ascends and increases as it descends. When this is combined with the changing speed, the result is a relatively constant centripetal acceleration (and force), making for a better ride for the passengers. 2
2. The centrifugal force is used to explain the apparent force that acts on a object travelling with uniform circular motion. Consider turning a corner in a car. Your inertia would have you continue to move in a straight line at a constant speed. Instead, the force of static friction of the road on the car's tires makes the car move through the curve taking you with it. You feel as though you want to move outward from the centre of the curve when, in reality, this feeling stems from your inertia to move straight forward although this direction is constantly changing. The centrifugal force does not really exist. Instead, it is used to account for the apparent force acting on an object within a noninertial frame of reference. 2
3. The force of gravity is directly proportional to the mass of the object it is being exerted upon.
The force of gravity varies inversely as the square of the distance between the centres of mass of the two objects.

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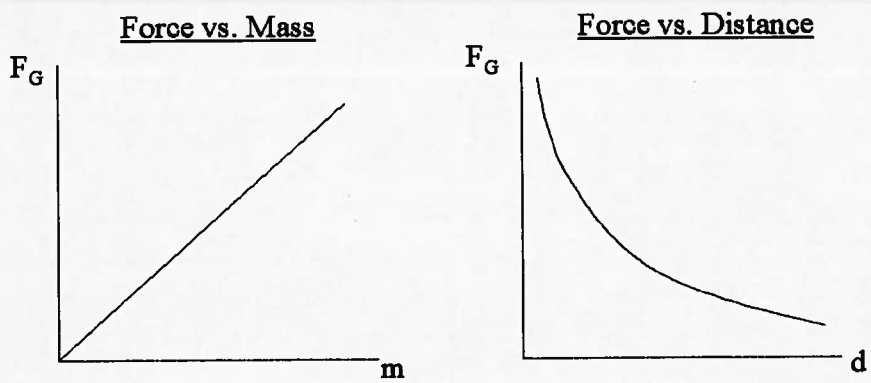

4. Since the acceleration due to gravity varies in the same ways as the force of gravity, its value on the surface of Earth is directly proportional to the mass of Earth and inversely proportional the square of the planet's radius. If both these quantities were twice their present values, the acceleration due to gravity would be one-half its present value. The effect of doubling the mass would double the acceleration, and the effect of doubling the radius would reduce the acceleration by a factor of four. Combining these effects results in the acceleration due to gravity being reduced by a factor of two.

$g_2 = \frac{g_1}{2}$

5. The law of universal gravitation applies only to two spherical objects (e.g., Earth and the Moon), to two objects whose centres are separated by a distance much greater than their size, or to a small object and a very large object (e.g., a person and Earth).

2 any two

$F = \frac{Gm_1m_2}{r^2}\left(\frac{2}{4}\right)$

- spherical
- distance for greater size
- small and large

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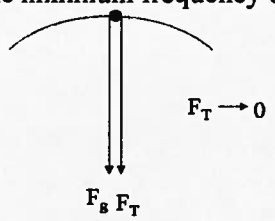
7. (a)
The minimum frequency occurs when the tension becomes zero.



$$F_C = F_g$$

$$m4\pi^2 Rf^2 = mg$$

$$f = \sqrt{\frac{g}{4\pi^2 R}}$$

$$= \sqrt{\frac{9.8\text{ N/kg}}{4\pi^2(1.6\text{ m})}}$$

$$f = 0.39\text{ Hz}$$

The minimum frequency is 0.39 Hz.

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(b)
The maximum tension occurs at the bottom of the circle.

Let "up" be negative and "down" be positive:

$$\vec{F}_C = \vec{F}_T + \vec{F}_g$$

$$\vec{F}_T = \vec{F}_C - \vec{F}_g$$

$$= -m4\pi^2 Rf^2 - mg$$

$$= -0.200\text{ kg}(4\pi^2)(1.6\text{ m})(0.39\text{ Hz})^2 - 0.200\text{ kg}(9.8\text{ N/kg})$$

$$\vec{F}_T = -3.9\text{ N}$$

The maximum tension is 3.9 N [up].

8. (a)
The minimum centripetal acceleration occurs when the frequency of rotation is a minimum.

minimum $a_C = 4\pi^2 Rf^2$

$= 4\pi^2 (4.0 \times 10^{-2}\ \text{m})(3.5\text{Hz})^2$

$a_C = 19\ \text{m/s}^2$

The maximum centripetal acceleration occurs when the frequency of rotation is a maximum.

maximum $a_C = 4\pi^2 Rf^2$

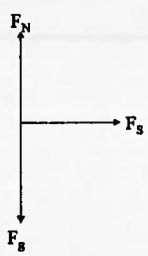
$= 4\pi^2 (4.0 \times 10^{-2}\ \text{m})(8.0\ \text{Hz})^2$

$a_C = 1.0 \times 10^2\ \text{m/s}^2$

$a_C = 1.0 \times 10^2\ \text{m/s}^2$

The insect's minimum centripetal acceleration is 19 m/s² and its maximum centripetal accelerations is 1.0 × 10² m/s².

(b)
The free-body diagram of the insect on the disc:
(F_C is supplied by static friction F_S)



$F_S = F_C$

$= ma_C$

$= 5.0 \times 10^{-4}\ \text{kg}(19\text{m/s}^2)$

$F_S = 9.67 \times 10^{-3}\ \text{N}$

$F_S \le \mu_S F_N$

$\mu_S \ge \frac{F_S}{F_N}$

$\mu_S \ge \frac{F_S}{mg}$

$\mu_S \ge 2.0$

$$F_S \le \mu_S F_N$$

$$\mu_S \ge \frac{F_S}{F_N}$$

$$\ge \frac{F_S}{F_g}$$

$$\ge \frac{F_S}{mg}$$

$$\ge \frac{9.67 \times 10^{-3}\ \text{N}}{5.0 \times 10^{-4}\ \text{N}(9.8\ \text{N/kg})}$$

$$\mu_S \ge 2.0$$

The minimum value of the coefficient of static friction is 2.0.

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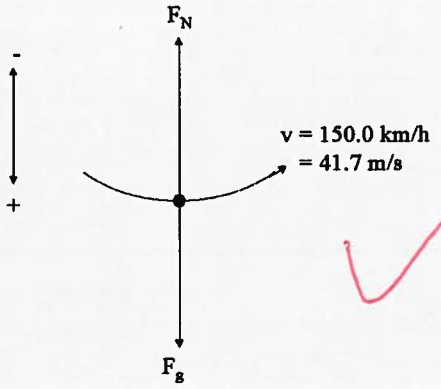
9. The free body diagram of the pilot at the bottom of the arc:



F_N = force of seat exerted upward on the pilot (the normal force)

$F_N = 4mg$

$$\vec{F}_C = \vec{F}_N + \vec{F}_g$$

$$\frac{-mv^2}{R} = -4mg + mg$$

$$\frac{mv^2}{R} = 3mg$$

$$R = \frac{v^2}{3g}$$

$$= \frac{(41.7 \text{ m/s})^2}{3(9.8 \text{ N/kg})}$$

$$R = 59 \text{ m}$$

The radius of the arc is 59 m.

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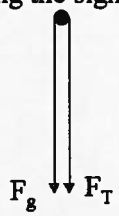
13. (a)
Using the sign convention "up" is negative and "down" is positive:



At the top of the circle and at minimum speed, $F_T = 0.0$ N.

$$\vec{F}_C = \vec{F}_g$$

$$\frac{mv^2}{R} = mg$$

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$$v = \sqrt{Rg}$$

$$= \sqrt{1.2\text{ m}(9.8\text{ N/kg})}$$

$$v = 3.4\text{ m/s}$$

The ball must have a minimum speed of 3.4 m/s to stay in a circular path.

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(b)
The maximum tension is achieved at the bottom of the circle:

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$$\vec{F}_C = \vec{F}_g + \vec{F}_T$$

$$\vec{F}_T = \vec{F}_C - \vec{F}_g$$

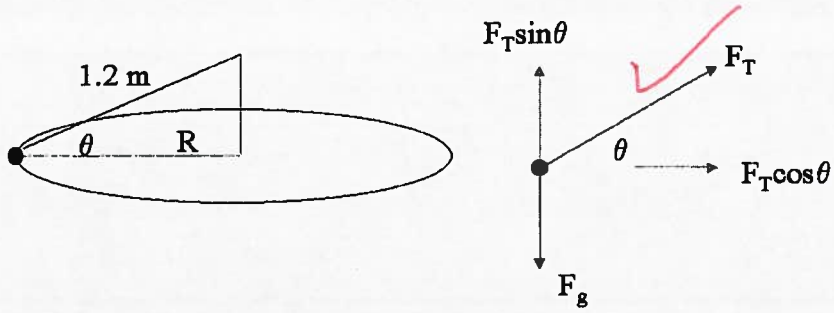
$$= -\frac{mv^2}{R} - mg$$

$$= -\frac{4.0\text{ kg}(3.43\text{ m/s})^2}{1.2\text{m}} - 4.0\text{ kg}(9.8\text{ N/kg})$$

$$\vec{F}_T = -78\text{ N}$$

The maximum tension in the rope is 78 N.

(c)



The centripetal acceleration (and force) are directed toward the centre of the circle. This means that the two vertical forces are balanced.

$F_g = F_T \sin\theta$

$$F_T = \frac{F_g}{\sin\theta}$$

In the horizontal plane, the centripetal force is supplied by the horizontal component of the tension:

$F_C = F_T \cos\theta$

$$F_T = \frac{F_C}{\cos\theta}$$

Combining the two expressions:

$$\frac{F_C}{\cos\theta} = \frac{F_g}{\sin\theta}$$

$$\frac{F_g}{F_C} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{mg}{\left(\frac{mv^2}{R}\right)} = \tan\theta$$

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$$\tan\theta = \frac{Rg}{v^2}$$

$$\theta = \tan^{-1}\left(\frac{Rg}{v^2}\right)$$

$$= \tan^{-1}\left(\frac{1.2\text{ m}(9.8\text{ N/kg})}{(3.43\text{ m/s})^2}\right)$$

$$\theta = 45°$$

The string makes an angle of 45° to the horizontal.

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14. At Earth's surface:

$F_g = mg$

$= 80.0\ \text{kg}(9.8\ \text{N/kg})$

$F_g = 784\ \text{N}$

Since $F_g \propto \frac{1}{r^2}$, then $F_g(r^2)$ is a constant.

If F_1 = force at Earth's surface
r_1 = Earth's radius
F_2 = force at position in question
$r_2 = 2.5r_1 + r_1 = 3.5r_1$
$F_1(r_1)^2 = F_2(r_2)^2$

$$F_2 = \frac{F_1 r_1^2}{r_2^2}$$

$$= \frac{784\ \text{N}(r_1^2)}{(5.5r_1)^2}$$

$F_2 = 2.6 \times 10^1\ \text{N}$

Earth exerts a force of 2.6×10^1 N on the astronaut.

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15. The weight of a 60.0-kg person at Earth's surface:
$F_g = mg$
$= 60.0\ \text{kg}(9.8\ \text{N/kg})$
$F_g = 588\ \text{N}$

Since $F_g = \dfrac{Gm_1 m_2}{r^2}$, the two planets can be compared.

Earth: $F_E = 588\ \text{N}$
$m_1 = 60.0\ \text{kg}$
$m_2 = m_E$
$r = r_E$

Planet: $F_P = ?$
$m_1 = 60.0\ \text{kg}$
$m_2 = 2.5\ m_E$
$r_P = 1.2 r_E$

3

$$\frac{F_P}{F_E} = \frac{\left(\dfrac{Gm_1(2.5m_E)}{(1.2r_E)^2}\right)}{\left(\dfrac{Gm_1 m_E}{(r_E)^2}\right)}$$

$$F_P = 588\ \text{N}\left(\frac{2.5m_E(r_E)^2}{m_E(1.2r_E)^2}\right)$$

$$F_P = 1.0 \times 10^3\ \text{N}$$

The person would weigh 1.0×10^3 N at the planet's surface.

16. $F_g = mg$ where $g = 3.4$ N/kg and
$F_g = \dfrac{GMm}{r^2}$ where M = mass of the planet and r = radius of the planet.

Both expressions represent the same force of gravity acting on mass m.

$$mg = \frac{GMm}{r^2}$$

$$r = \sqrt{\frac{GM}{g}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\ (7.2 \times 10^{22}\ \text{kg})}{3.4\ \text{N/kg}}}$$

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$$r = 1.2 \times 10^6\ \text{m}$$

The planet's radius is 1.2×10^6 m.

17. The orbital radius is 6.38×10^6 m + 3.25×10^5 m = 6.705×10^6 m
The centripetal force acting on the satellite is supplied by gravity.
$F_C = F_g$

$$\frac{m4\pi^2 R}{T^2} = \frac{GM_E m}{R^2}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$$

$$= \sqrt{\frac{4\pi^2 (6.705 \times 10^6 \text{ m})^3}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 (5.98 \times 10^{24} \text{ kg})}}$$

$$T = 5.46 \times 10^3 \text{ s}$$

$$\frac{5.46 \times 10^3 \text{ s}}{60 \text{ s/min}} = 91.0 \text{ min}$$

The orbital period is 91.0 min.

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18. The centripetal force acting on the satellite is supplied by gravity.
$F_C = F_g$

$$\frac{m4\pi^2 R}{T^2} = \frac{GM_E m}{R^2}$$

$$R = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}}$$

$$= \sqrt[3]{\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 (5.98 \times 10^{24} \text{ kg}) \left((3.2 \text{ h})(3.6 \times 10^3 \text{ s/h})\right)^2}{4\pi^2}}$$

$$R = 1.1 \times 10^7 \text{ m}$$

The orbital radius is 1.1×10^7 m.

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19. The centripetal force acting on the satellite is supplied by gravity.

$F_C = F_g$

$$\frac{mv^2}{R} = \frac{GM_E m}{R^2}$$

$$R = \frac{GM_E}{v^2}$$

$$= \frac{6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\ (5.98 \times 10^{24}\ \text{kg})}{\left(4.2 \times 10^3\ \text{m/s}\right)^2}$$

$$R = 2.26 \times 10^7\ \text{m}$$

$2.26 \times 10^7\ \text{m} - 6.38 \times 10^6\ \text{m} = 1.6 \times 10^7\ \text{m}$

The altitude above the Earth's surface is 1.6×10^7 m.

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